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An Optimized Advance Detector Configuration for Option Zone Protection at High Speed Signalized Intersections

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ABSTRACT

Advance detection and green extension schemes are widely applied in practice as a typical solution to the safety issues associated with the intersection dilemma zone (DZ) problem. Most existing detector configurations were either developed based on the traditional Type I DZ model in which some critical contributing factors were assumed static, or based on generic Type II DZ. A comparison analysis based on field-observed trajectory data showed that the option zone model estimated the location of dilemma zone most accurately among all available dilemma zone models. The authors' recent research on the quantitative modeling of option zone's contributing factors made it possible to accurately identify the option zone locations. That lays out a solid foundation for developing an option zone-based detection scheme in order to achieve the most effective and efficient dilemma zone protection. This paper presents an alternative advance detector configuration for option zone protection via optimization trials within a calibrated VISSIM simulation model. The optimization objective was to minimize the combined cost of dilemma hazard (safety) and delay (mobility). Dilemma Conflict Potential, a comprehensive dilemma hazard model was used to quantitatively measure the safety performance, as a replacement for the traditional measure of "number of vehicles in dilemma zone". The optimal configuration was evaluated and validated via its comparison with four widely-applied detector configurations in the nation. The results revealed the superiority of the developed optimal detector configuration in terms of the best safety performance and the least combined cost of dilemma hazard and delay among all configurations.

INTRODUCTION

As a major cause of rear-end and right-angle crashes at high speed signalized intersections, dilemma zone is regarded as one of the most critical intersection safety issues that have not been fully solved yet (*1*). To address the safety issue caused by dilemma zone, the most economical and widely applied solution is placing advance point detectors (e.g., small-area inductive loop detectors) in advance of the dilemma zones (*2*). With the advance detection, vehicles can be detected during the course when they approach the intersection. Extended green time was hence given by the controller to clear these vehicles out of the dilemma zone before the signal's transition to yellow indication. When developing an advance detector configuration, there is always a trade-off between the safety performance and the intersection's operational efficiency. Extending the green time on the major road will cause a longer delay on the minor street, as well as a longer cycle length, which will degrade the overall operational efficiency of the intersection. Therefore, how to achieve a balance between safety and operational efficiency is of great interest for practitioners and researchers.

FHWA's Traffic Detector Handbook (*2*) gives guidelines for advance detector's placement, including Winston-Salem configuration (*2*), SSITE configuration (*3-4*), and Beirele configuration (*5*). However, most of these configurations were developed before 1980, when there was a lack of standard measures of effectiveness and powerful simulation tools for quantitatively evaluating the safety and operational performance of these detection systems. The dilemma zone locations used to develop these configurations, such as Type I dilemma zone estimated using static parameters (*2, 5*) and based on engineering judgment (*3-4*), were not comparable to the actual locations of dilemma zone of today due to the rapid advance in vehicular technology and change in driver behavior during the past few decades.

In summary, it has been more than thirty years since those advance detector configurations recommended by FHWA were developed. The location of dilemma zone has already changed (*17, 24, 25*). Regarding these facts, an updated advance detector configuration based on the updated dilemma zone location is highly demanded. In this context, this paper is dedicated to developing an updated advance detector configuration, which is based on the recent findings of locations of dynamic option zones. Moreover, the proposed detector configuration is to be optimized by maximizing the combined safety and operational performance using the state-of-the-art traffic simulation tools.

LITERATURE REVIEW

There are two types of dilemma zone (i.e., Type I and Type II) with completely different definitions. The Type I dilemma zone was defined by Gazis et al. as a zone in which at the onset of yellow indication the driver can neither clear the intersection during the yellow interval nor safely stop before the stop line (*6*). A longer yellow interval could eliminate the Type I dilemma zone. However, it would produce a longer option zone at the same time (*7-9*). The option zone is the zone in which vehicles can either pass the intersection during the yellow time or safely stop before the stop line. Researchers found option zone is also hazardous, because drivers in option zone are also very likely to get involved in rear-end and right angle accident (*7, 8*). This fact suggests that protection should be given to both Type I dilemma zone and option zone. The locations of Type I dilemma zone and option zone are determined by the minimum stopping distance (X_c) and the maximum yellow-light-running distance (X_0) (*6*). When X_c is greater than X_0, the Type I dilemma zone forms. When X_0 is greater than X_c, the option zone exists.

Mathematically, X_0 and X_c are mainly contributed by driver's minimum perception-reaction time (PRT), maximum deceleration rate for stopping, and, maximum acceleration rate for running. Due to the lack of qualitative knowledge about these factors, they are typically assumed to have constant nominal values in most practices (*2, 10, 11*).

This lack of such quantitative knowledge about the contributing factors prevented accurate computations of dilemma zone. To overcome this problem, Zegeer proposed another definition of dilemma zone using driver's stopping probability in response to the yellow indication (*12*). His dilemma zone was defined as the road segment where more than 10% and less than 90% of the drivers would choose to stop. This definition was further known as the Type II dilemma zone or indecision zone (IZ) (*13-16*). The advantage of Type II dilemma zone is that it can be easily computed through obtaining the driver's stopping probability model from binary logistic regression analysis.

Recent research conducted by the authors investigated real-world drivers' acceleration and deceleration behavior in response to the yellow indication. The results revealed that the contributing factors to Type I dilemma zone and option zone are dynamic rather than static (*17*). They also established numerical models of these contributing factors as functions of vehicle's speed and intersection approach's 85th percentile speed. With the quantitative knowledge of these contributing factors, the traditional Type I dilemma zone and option zone models were hence updated to reflect more accurate locations of the zones. The dynamics of dilemma zone was also explored in other research (*26, 27*).

Due to the lack of standard measures of effectiveness for quantitatively evaluating the safety and operational performance, advance detector configurations proposed in early time (before 1980) were mostly developed without optimization (*2-5*). In 1993, Bonneson and McCoy introduced the use of maximum allowable headway (MAH) as a quantitative measure of effectiveness for evaluating the advance detection system (*18*). They found small-MAH design is supposed to have lower max-out frequency as well as lower overall delay. Recently, Bonneson and Pratt continued their work by proposing a practical framework for evaluating the advance detection configurations (*19*). The framework was useful for determining the optimal passage time for achieving a high safety performance.

In 2009, Li and Abbas started to use simulation tool for optimizing the advance detector configuration (*20*). A traffic simulation program developed by them was used as the simulation test bed. Their proposed advance detector configuration was based on the Type II dilemma zone model, and was optimized using genetic algorithm whose objective is to minimize both the dilemma zone cost and the delay cost. Specifically, a new traffic-conflict-based safety measure, called the dilemma hazard, was used to evaluate the safety performance. The dilemma hazard model in their research was calibrated using simulation.

USE OF OPTION ZONE MODEL IN ESTIMATING DILEMMA ZONE

Most advance detector configurations were developed based on either traditional (static) Type I dilemma zone model (*2, 5*) or generic Type II dilemma zone model (*3, 4, 6, 16, 20*) which assumes fixed dilemma zone boundaries (e.g., 2 to 5 seconds from the stop line). Some states like Minnesota, uses 5.5 seconds from the stop line to define the upstream boundary of Type II dilemma zone when developing their advance detection scheme. The potential use of option zone model has not been discussed in the literature. In fact, there is a lack of literature documenting a comparison between alternative dilemma zone models in terms of the accuracy in estimating the actual dilemma zone. In this research, an accuracy indicator for dilemma zone estimation is introduced, which is the root-mean-squared-error (RMSE) for the horizontal distance between the boundary curve of an alternative dilemma zone model and the most closely stopped vehicles or the furthest yellow-light-running vehicles. The yellow-light-running vehicle is defined as a vehicle entering the intersection on yellow. The RMSE can be calculated using the following equation.

$$RMSE = \sqrt{\frac{\sum_{i}^{n}[X_{DZ-bound}(V_i) - X_i]^2}{n}} \qquad (1)$$

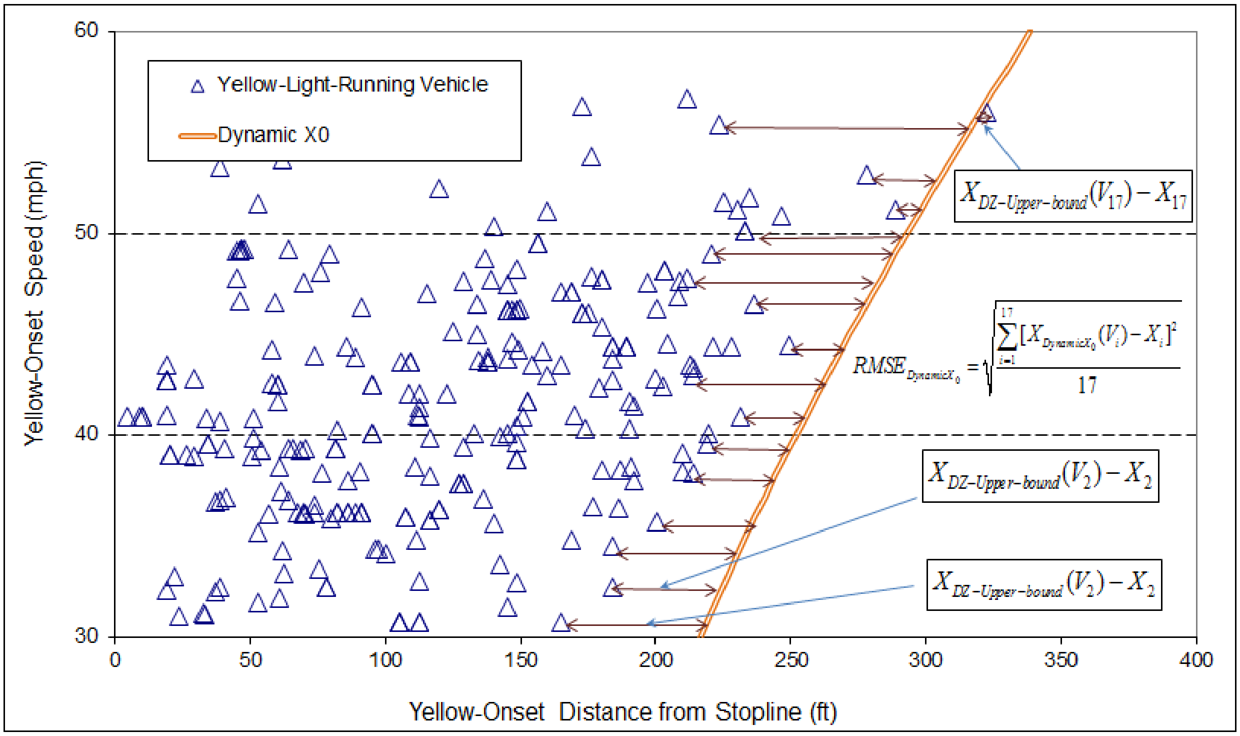
Where, RMSE = the root-mean-squared-error;

$X_{DZ\text{-}bound}(V_i)$ = the boundary computed by the alternative dilemma zone model (ft);

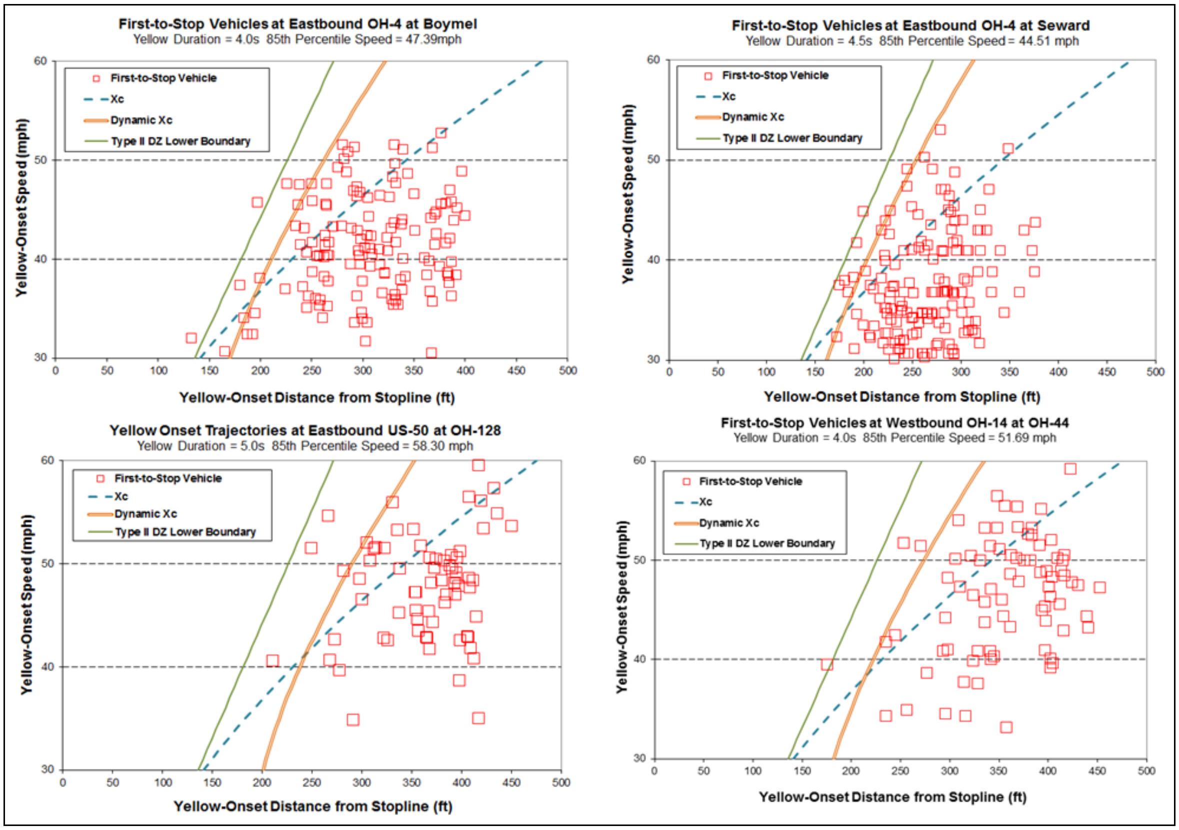
X_i = the observed minimum stopping distance or maximum yellow passing distance (ft); and,

n = the number of all observed minimum stopping distance or maximum yellow passing distances.

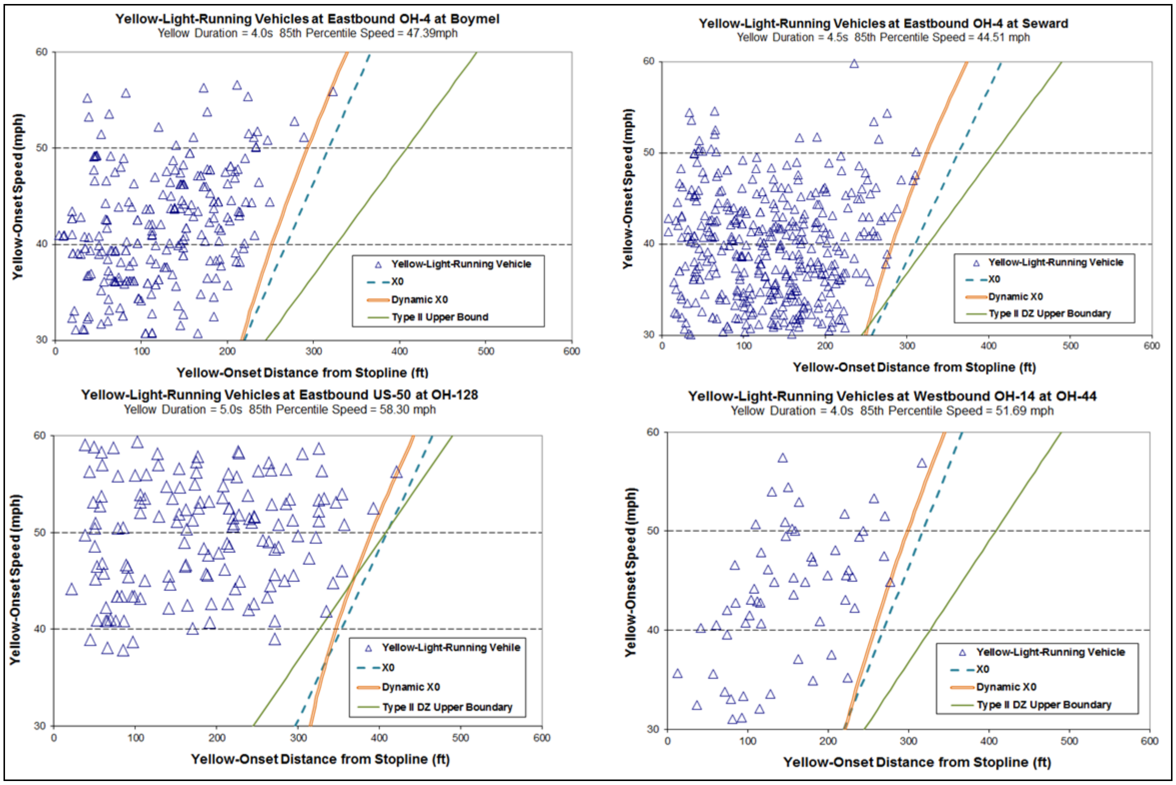
Figure 1.a shows an example of the calculation of RMSE for upper boundary (X_0) of the option zone model.



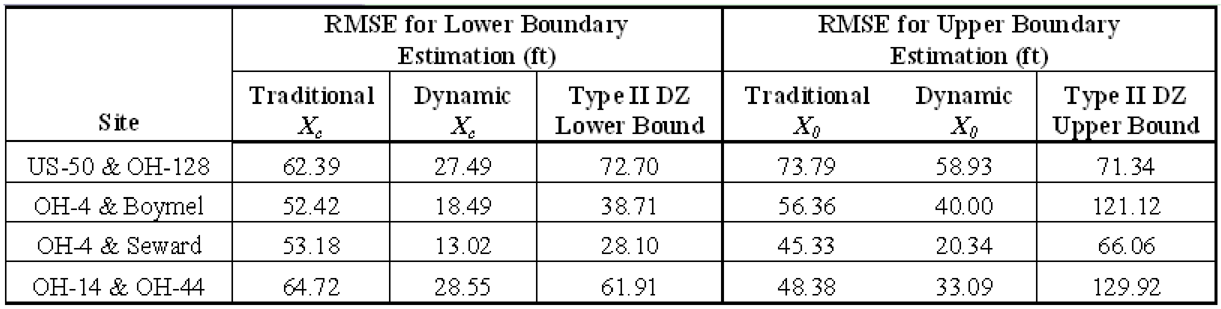
(a)


First-to-Stop Vehicles at Eastbound OH-4 at Boymel
Yellow Duration = 4.0s 85th Percentile Speed = 47.39mph
First-to-Stop Vehicles at Eastbound OH-4 at Seward
Yellow Duration = 4.5s 85th Percentile Speed = 44.51 mph
Yellow Onset Trajectories at Eastbound US-50 at OH-128
Yellow Duration = 5.0s 85th Percentile Speed = 58.30 mph
First-to-Stop Vehicles at Westbound OH-14 at OH-44
Yellow Duration = 4.0s 85th Percentile Speed = 51.69 mph
First-to-Stop Vehicle
Xc
Dynamic Xc
Type II DZ Lower Boundary
Yellow-Onset Speed (mph)
Yellow-Onset Distance from Stopline (ft)
30
40
50
60
0
50
100
150
200
250
300
350
400
450
500

(b)


Yellow-Light-Running Vehicles at Eastbound OH-4 at Boymel
Yellow Duration = 4.0s 85th Percentile Speed = 47.39mph
Yellow-Light-Running Vehicles at Eastbound OH-4 at Seward
Yellow Duration = 4.5s 85th Percentile Speed = 44.51 mph
Yellow-Light-Running Vehicles at Eastbound US-50 at OH-128
Yellow Duration = 5.0s 85th Percentile Speed = 58.30 mph
Yellow-Light-Running Vehicles at Westbound OH-14 at OH-44
Yellow Duration = 4.0s 85th Percentile Speed = 51.69 mph
Yellow-Light-Running Vehicle
Yellow-Light-Running Vehile
X0
Dynamic X0
Type II Upper Bound
Type II DZ Upper Boundary
Yellow-Onset Speed (mph)
Yellow-Onset Distance from Stopline (ft)
30
40
50
60
0
100
200
300
400
500
600

(c)



Site	RMSE for Lower Boundary Estimation (ft)			RMSE for Upper Boundary Estimation (ft)		
	Traditional X_c	Dynamic X_c	Type II DZ Lower Bound	Traditional X_0	Dynamic X_0	Type II DZ Upper Bound
US-50 & OH-128	62.39	27.49	72.70	73.79	58.93	71.34
OH-4 & Boymel	52.42	18.49	38.71	56.36	40.00	121.12
OH-4 & Seward	53.18	13.02	28.10	45.33	20.34	66.06
OH-14 & OH-44	64.72	28.55	61.91	48.38	33.09	129.92

(d)

FIGURE 1 Accuracy in estimating the actual dilemma zone.

According to its statistical implication, the smaller the RMSE is, the better the boundary curve of the alternative dilemma zone model fits the observed maximum yellow-light-running distances or minimum stopping distances. Therefore, a smaller RMSE reflects a higher accuracy for estimation of the actual dilemma zone. Figures 1.b and 1.c compare the lower and upper boundaries of the traditional dilemma zone (computed using assumed constant contributing factor values recommended by ITE, i.e. 1s for PRT, 10 ft/s^2 for deceleration rate), the dynamic option zone, and the Type II DZ (i.e., 3.08-5.56s travel time from stop line obtained from logistic regression based on observed trajectory data from all four study intersections). The RMSEs for different dilemma zone models are compared in Figure 1.d. The results indicate that boundaries of the dynamic option zone have the lowest RMSE at all study sites. Results from ANOVA analysis further validated that the dynamic option zone model has a significantly lower RMSE or a significantly higher accuracy in estimating dilemma zone than the traditional DZ/option zone model and the Type II DZ model.

Based on these findings, the dynamic option zone model was identified as the most appropriate dilemma zone model for developing the advance detection configuration because of it highest accuracy in estimating the dilemma zone boundary. The reason why the option zone model was not widely studied is probably the lack of quantitative knowledge about the contributing factors that determine the location of option zone. The authors' most recent research findings on the quantitative modeling of option zone's contributing factors make it possible to obtain an accurate option zone model (*17*). This has paved the road for developing an option zone based advance detector configuration, which can theoretically maximize the safety and operational performance of the detection system.

In this paper, an alternative advance detector configuration for option zone protection was therefore developed specifically based on the dynamic option zone model presented in the following section. The configuration was optimized to minimize the combined cost of the dilemma hazard and delay. Finally, the resulted optimal advance detector configuration was evaluated in terms of both safety and operational performance through comparison with other existing configurations that are widely used in the US.

RESEARCH METHODOLOGY

The entire methodology for developing the optimal advance detector configuration for option zone protection is illustrated by Figure 2. The configuration was specifically developed based on the updated dynamic option zone model. The optimization process was conducted in a calibrated VISSIM traffic simulation test bed. During the optimization, the safety performance was measured by "Dilemma Conflict Potential" (DCP), a dilemma hazard model proposed in this research. DCP computed the probability of rear-end and right-angle traffic conflicts faced by each approaching vehicle based the vehicle's and its leading vehicle's speed and location at the onset of yellow time. The DCP model was calibrated using field-collected trajectory data. On the other hand, the operational efficiency was measured by the overall intersection delay obtained from VISSIM output. During the simulation, the traffic volumes were designed to vary with different periods of the day (24 hours). The objective function of the optimization was to minimize the daily combined cost of dilemma hazard and delay. In the end, the resulted optimal configuration was compared with four classic detector configurations: Bonneson, Beirele, SSITE, and Winston-Salem configurations (*2, 16*), in terms of safety performance and operational efficiency.

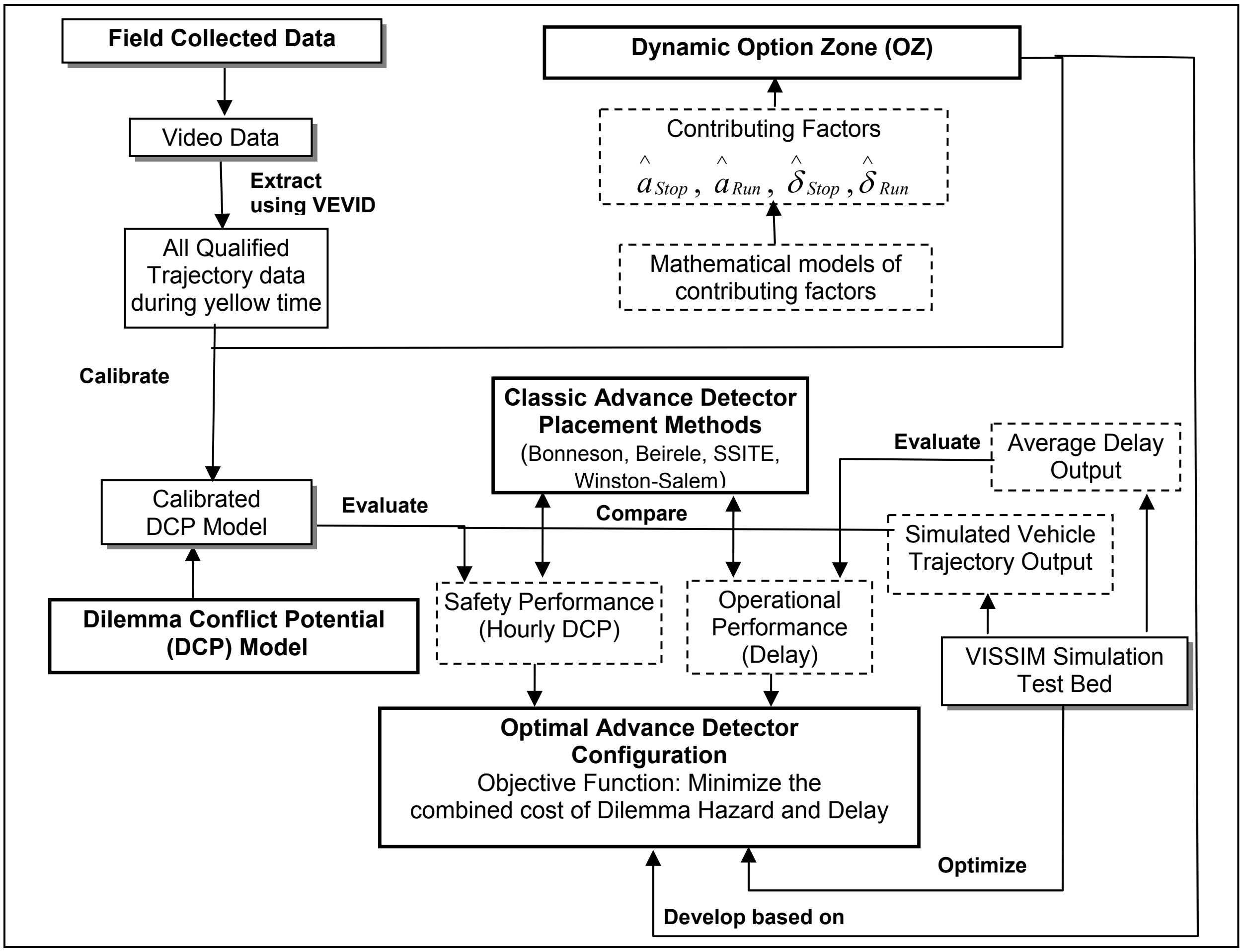


FIGURE 2 Illustration of the research methodology.

DYNAMIC OPTION ZONE MODEL

The authors' recent research revealed the dynamic natures of the contributing factors (i.e., the minimum PRT for stopping $\hat{\delta}_{Stop}$, the maximum deceleration rate for stopping $\hat{a}_{Stop}$, the maximum acceleration rate $\hat{a}_{Run}$ for running, and the minimum PRT for running $\hat{\delta}_{Run}$) to Type I dilemma zone and option zone (17). The traditional Type I dilemma zone and option zone model was therefore updated to reflect the dynamic features of the contributing factors. The following equations are the updated mathematical form for the Type I dilemma zone and option zone model.

$$X_c(V_0,V_{85th}) = V_0\,\hat{\delta}_{stop}(V_0) + \frac{V_0^{\,2}}{2\cdot \hat{a}_{stop}(V_0,V_{85th})} \qquad (2)$$

$$X_0(V_0,\tau,V_{85th}) = V_0\tau + \frac{1}{2}\hat{a}_{Run}(V_0,V_{85th})\cdot[\tau - \hat{\delta}_{Run}(V_0)]^2 \qquad (3)$$

Where, V_0 = vehicle's approaching speed (ft/s);
V_{85th} = 85th percentile speed of the intersection approach (ft/s);
$X_c(V_0, V_{85th})$ = critical (minimum) stopping distance from stop line at speed V_0 and under 85th percentile speed V_{85th} (ft);
$X_0(V_0, V_{85th})$ = maximum yellow light running distance from stop line at speed V_0 and under 85th percentile speed V_{85th} (ft);
$\hat{\delta}_{stop}(V_0, V_{85th})$ = minimum PRT for stopping at speed V_0 and under 85th percentile speed V_{85th} (s);
$\hat{a}_{stop}(V_0, V_{85th})$ = maximum deceleration rate for stopping at speed V_0 and under 85th percentile speed V_{85th} (ft/s^2);
$\hat{\delta}_{Run}(V_0, V_{85th})$ = minimum PRT for yellow light running at speed V_0 and under 85th percentile speed V_{85th} (s);
$\hat{a}_{Run}(V_0, V_{85th})$ = maximum acceleration rate for yellow light running at speed V_0 and under 85th percentile speed V_{85th} (ft/s^2).

In Equations (2) and (3), the contributing factors are functions of vehicle's speed and intersection approach's 85th percentile speed. Specifically they are represented by the following equations:

$$\hat{\delta}_{Stop}(V_0) = \hat{\delta}_{Run}(V_0) = 0.445 + \frac{21.478}{V_0} \qquad (4)$$

$$\hat{a}_{stop}(V_0,V_{85th}) = \exp(3.379 + \frac{-36.099}{V_0}) - 9.722 + \frac{429.692}{V_{85th}} \qquad (5)$$

$$\hat{a}_{Run}(V_0,V_{85th}) = -27.91 + \frac{760.258}{V_0} + 0.266\cdot V_{85th} \qquad (6)$$

When $X_c > X_0$, the Type I dilemma zone is formed. In the case of $X_c < X_0$, the Type I dilemma zone is eliminated, and the roadway segment between X_0 and X_c is the option zone. Previous research also revealed that only option zone exists when the yellow interval is equal or greater than 4.0 sec, while the Type I dilemma zone is completely eliminated by the long yellow interval (21). In other words, when the yellow interval is greater than 4.0 sec, protection is only needed for option zone. Therefore, in this paper, the proposed alternative advance detector configuration was specially designed for option zone protection due to the fact that most high speed intersections have a yellow interval greater than 4.0 sec.

ALTERNATIVE ADVANCE DETECTOR CONFIGURATION FOR OPTION ZONE PROTECTION

In this research, the goal of the proposed advance detector configuration was twofold: (1) firstly, to assure the safety of all vehicles traveling in the protected speed range; (2) secondly, to maximize the operation efficiency after the safety is guaranteed. Based on this goal, the following design criteria were established for developing the alternative advance detector configuration. (See Figure 3)

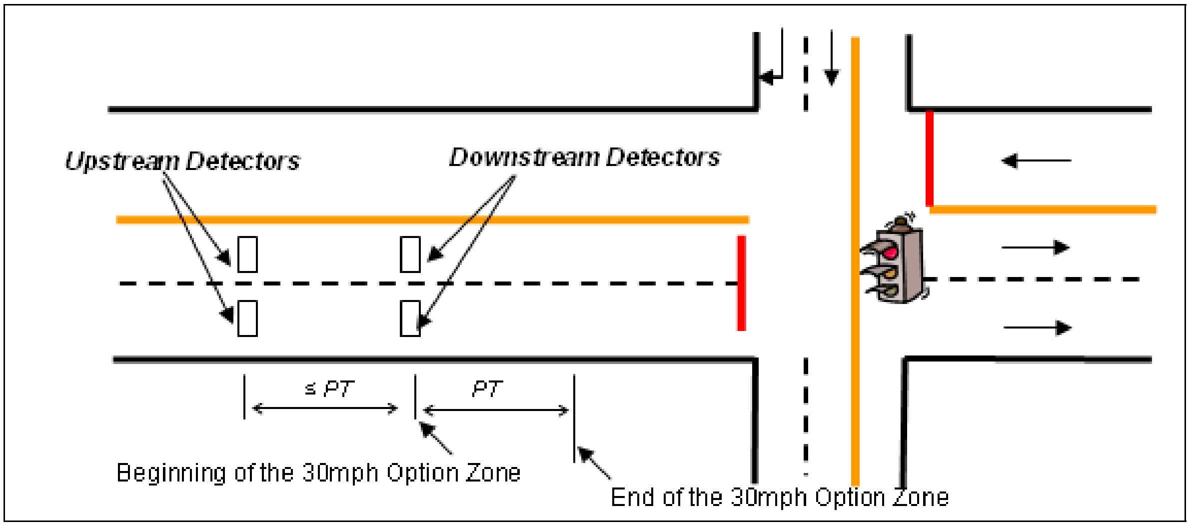


FIGURE 3 Illustration of design criteria for the alterntive advance detector configuration.

- The design should be based on the dynamic option zone rather than the static option zone or the Type II dilemma zone based on the findings that the dynamic option zone model estimates the actual dilemma zone most accurately among all the three available dilemma zone models.
- Two advance detectors should be used in the design: upstream and downstream detectors.
- The lowest protected speed should be 30mph, because vehicles traveling below 30mph can easily manage a safe stop in response to yellow indications. Namely, the downstream detector should be placed at the beginning of the 30mph option zone.
- The passage time should be the minimum required time for carrying vehicles traveling at the lowest protected speed, i.e., 30mph, from the downstream detector to the end of the 30mph option zone rather than to the stop line. This could relatively reduce the passage time in order to maximize the operational efficiency.
- The highest protected speed should be no less than the posted speed limit of the intersection approach. In other words, the upstream detector should be placed at or upstream before the beginning of the option zone of the posted speed limit. (e.g., for an approach having a 50mph speed limit, the upstream detector should be placed at or upstream before the beginning of the 50mph option zone.)
- The travel time between the two advance detectors should be no longer than the passage time. In other words, the furthest possible position of the upstream detector should be located at the passage time (e.g., 2 seconds) from the downstream detector. And, the highest possible protected speed should be determined by the location of the upstream detector. Note that the travel time is calculated based on 30mph. This can guarantee that full protection is given to all vehicles whose traveling speeds are greater than 30mph and less than the highest protected speed.
- According to the above two criteria, the location of the upstream detector should fall into the interval [beginning of the option zone of the posted speed limit, the passage time from the downstream detector]. Its final location should be determined after optimization.

OPTIMIZATION OF THE ALTERNATIVE ADVANCE DETECTOR CONFIGURATION

Dilemma Conflict Potential as a Measure of Effectiveness for Safety

As a replacement for the traditional measure of "number of vehicles in dilemma zone", a new concept of dilemma Conflict Potential (DCP) was proposed in this research to measure the dilemma hazard faced by each vehicle. DCP is defined as the probability for an approaching vehicle to have potential traffic conflicts associated with dilemma zone. Typically, the dilemma zone can result in two major types of traffic conflicts: rear-end (RE) conflict and right-angle (RA) conflict. A rear-end conflict occurs when the vehicle ahead of the target vehicle stops abruptly while the target vehicle intends to go. A right-angle conflict takes place when the leading vehicle chooses to go while the target vehicle attempts to run red. In this context, the DCP model was designed to address the probability of both right-angle and rear-end conflicts.

TABLE 1 Possible Dilemma Conflict Scenarios and the Corresponding DCP

Scenario	Target Vehicle's Position at the Yellow Onset	Lead Vehicle's Position at the Yellow Onset	DCP
1	in Type I DZ	any position / none	1
2	in option zone	in option zone	$DCP_{S2}(RE)+DCP_{S2}(RA)$
3	in option zone	in Type I DZ	$DCP_{S3}(RE)+DCP_{S3}(RA)$
4	in option zone	not in any zone / none	$DCP_{S4}(RA)$
5	not in any zone	in Type I DZ	$DCP_{S5}(RE)$
6	not in any zone	in option zone	$DCP_{S6}(RE)$
7	not in any zone	not in any zone	0

Numerically, the DCP model computes vehicle's conflict probability based on the vehicle's and its leading vehicle's speed and location at the onset of yellow indication. Seven mutually exclusive and collectively exhaustive scenarios at the onset of yellow indication were considered for modeling DCP. They cover all possible situations that may lead the target vehicle to a potential rear-end or right-angle conflict. The computation for the DCP for each scenario is summarized in Table 1. The DCP were mathematically modeled based on the conditional probability for a vehicle to have traffic conflicts given the vehicle's maneuver in response to the yellow indication. The DCP models were calibrated using field-collected trajectory data through the calibration of these probability models. Detailed modeling and calibration process of the DCP model can be found in one of the authors' research reports (*21*).

Objective Function and Constraint of the Optimization

The optimization aimed at obtaining a configuration of detectors, which can maximize both safety performance and operational efficiency. The safety performance was assessed by the total number of dilemma zone related traffic conflicts per hour ($C_{\text{Hourly-Total}}$). A smaller $C_{\text{Hourly-Total}}$ reflects a better safety performance. As introduced in the previous subsection, DCP is the probability for an approaching vehicle to have dilemma zone related traffic conflicts. Therefore, the summation of the DCPs of all vehicles traveling on the main street for one hour represents the total number of the dilemma zone related traffic conflicts per hour. This relationship can be expressed by the following equation.

$$C_{Hourly-Total} = \sum_{i}^{1-hour} DCP_i \tag{7}$$

Where, $C_{\text{Hourly-Total}}$ = total number of DZ related traffic conflicts per hour (conflict);
DCP_i = dilemma conflict potential for the approaching vehicle i (conflict).

The operational efficiency was assessed by the overall intersection delay (D_{Overall}). A smaller D_{Overall} reflects a better operational efficiency. The overall intersection delay was the weighted average delay of all movements, which can be quantitatively represented by the following equation.

$$D_{Overall} = \sum_i D_i \cdot Q_i \Big/ \sum_i Q_i \tag{8}$$

Where, D_{Overall} = overall intersection delay per vehicle (sec/veh);
D_i = the delay for movement i (sec/veh);
Q_i = the hourly flow rate for movement i (veh/hr).

Considering that the measures of safety and operational efficiency had different units, it was difficult to make these two measurements comparable. Therefore, both of the measures were converted to US dollar in order to make them comparable. A previous study concluded that the probability for a traffic conflict to become a real accident was about 0.0001, while the average cost for each real accident was \$56,706 (*13*). Therefore, the unit cost of each traffic conflict was computed as \$56,706×0.0001, which is \$5.67 per conflict. The safety was hence measured in terms of money as the hourly dilemma conflict cost (\$), which can be mathematically represented by the following equation:

$$Cost_{Conflict-Hourly} = 5.67 \times C_{Hourly-Total} \tag{9}$$

Where, $Cost_{\text{Conflict-Hourly}}$ = hourly dilemma conflict cost (\$);

According to the US Bureau of Labor Statistics, the average hourly salary in the US was \$20.32/hr, which was equivalent to \$0.00564/sec (23). Using this unit cost, the operational efficiency was measured in terms of money, which was termed as the hourly delay cost (\$). Assuming that the driver was the only person in each vehicle, the hourly delay cost could then be represented by the following equation.

$$Cost_{Delay-Hourly} = 0.0056 \times D_{Overall} \times \sum_i Q_i \tag{10}$$

Where, $Cost_{\text{Delay-Hourly}}$ = hourly delay cost (\$);

Based on these aforementioned models, the objective of the optimization was eventually determined as minimizing the combined cost of dilemma conflicts and delay. Considering that the performance of the detection system may vary as traffic volumes vary during different hours of a day, the optimization objective was specifically defined as minimizing the daily combined cost of dilemma conflicts and delay. The objective function and the constraint are represented by the following equation.

$$Min(\sum_{i=1}^{24} Cost_{i-Conflicts-Hourly} + \sum_{i=1}^{24} Cost_{i-Delay-Hourly})$$

$$= F(n = 2,\ t = \frac{S_{begin}(30) - S_{end}(30)}{1.47 \times 30},\ x_{Downstream} = S_{begin}(30),\ x_{Upstream}) \qquad (11)$$

$$Subject\ to$$

$$S_{begin}(SpeedLimit) \le X_{Upstream} \le S_{begin}(30) + t \times 30 \times 1.47$$

Where, $Cost_{i\text{-}Confilcts\text{-}Hourly}$ = hourly dilemma conflict cost during the i^{th} hour of the day ($);
$Cost_{i\text{-}Delay\text{-}Hourly}$ = hourly delay cost for the i^{th} hour of the day ($);
n = number of advance detectors;
t = passage time (sec);
$S_{begin}(30)$ = location of the beginning of the 30mph option zone from stop line (ft);
$S_{begin}(30)$ = location of the end of the 30mph option zone measured from stop line (ft);
$X_{Downstream}$ = location of the downstream detector measured from stop line (ft);
$X_{Upstream}$ = location of the upstream detector measured from stop line (ft);

Simulation-based Optimization Test Bed

Microscopic traffic simulation software VISSIM was used in this research for performing the optimization. For each speed limit, a specific simulation model was built and calibrated. The signal controller used in the simulation is a NEMA controller operating at fully actuated mode. Standard dual-ring and two-barrier phasing design was used. To simplify the optimization, only two phases were designed: through phase on the mainline and through phase on the side street. The mainline through phases of both directions needed to cross the barrier at the same time, which indicates that a simultaneous gap-out strategy was used.

TABLE 2 Traffic Volume Setting for Different Hours of a Day

Very Low Volume	Low Volume	Moderate Volume	High Volume	Very High Volume
5 hours 0,1,2,3,4	5 hours 5,20,21,22,23	7 hours 6,10,11,12,13,14,19	5 hours 7,9,15,16,18	2 hours 8,17
Mainline: 150 veh/hr/ln[a] Side street: 100 veh/hr/ln	Mainline: 250 veh/hr/ln Side street: 200 veh/hr/ln	Mainline: 400 veh/hr/ln Side street: 300 veh/hr/ln	Mainline: 550 veh/hr/ln Side street: 500 veh/hr/ln	Mainline: 750 veh/hr/ln Side street: 650 veh/hr/ln

a. veh/hr/ln represents vehicles per lane per hour

The following parameters of the simulation model are calibrated using field observed data.

- Traffic composition: the traffic on both mainline and side street was composed of 90.3% cars and 9.7% heavy vehicles;
- Driver's stopping probability model: the parameter values were calibrated by performing binary logistic regression on the field collected yellow-onset trajectory data.

The traffic volume settings in the simulation model varied with the simulated time period of the day. The detailed settings of the traffic volumes are summarized in Table 2. Note that, the volumes listed in Table 2 are not actual field data, because specific locations would have site-specific distributions of volumes. They are assumed volumes based on a reasonable distribution in different periods of a day.

Optimization Process and Results

According to the objective function represented by Equation (11), the only variable in the optimization was the location of the upstream detector, which should fall into the interval [$S_{begin}(SpeedLimit)$, $S_{begin}(30)+t\times 30\times 1.47$]. Computed using the option zone model (Equation (3)), the candidate locations of the upstream detector were determined using 5 ft as the interval, as summarized in Table 3. The location of the downstream detector was fixed at the beginning of the 30mph option zone for a specific speed limit. Both upstream and downstream detectors are a small area detector with the size of 6 ft by 6 ft.

For each speed limit, a specific optimal configuration of detectors was generated from the optimization. Each candidate detector configuration was evaluated by 30 simulation runs for one specific volume condition. A total of 30×5 simulation runs were therefore required for each candidate detector configuration because five volume conditions were considered in the optimization. Each simulation run had a unique random seed, and lasted 600 simulation seconds, which equaled to 10 simulation minutes. The simulation resolution was set as 5 steps per simulation second. At each onset of yellow interval, the speed and location of all vehicles that were traveling on the mainline were exported to the database. The delay of each movement (e.g., eastbound mainline through) were exported as well at the end of each simulation run. Meanwhile, the termination status of the green time for each cycle (i.e., max-out or gap-out) was also exported.

A customized software program developed by the authors was used to access the outputted database and files by VISSIM. The program computed the Type I dilemma zone and option zone using Equations (2) and (3) for each vehicle based on the vehicle's yellow-onset speed, the 85th percentile speed of the intersection approach, and the duration of yellow interval. The program hence justified whether this vehicle was in option zone, in Type I dilemma zone, or not in any zone by examining the vehicle's yellow-onset location with its computed dilemma zone.

TABLE 3 Candidate Locations of Advance Detectors for Different Speed Limits

Speed Limit	Candidate Upstream Detector Location [c] (ft)	Downstream Detector Location [c] (ft)	PT [a] (s)	Travel Time between Two Detectors [b] (sec)
40 mph	243	209	1.4	0.77
	250	209		0.93
	255	209		1.04
	260	209		1.16
	265	209		1.27
	270	209		1.38
45 mph	289	236	1.7	1.20
	295	236		1.34
	300	236		1.45
	305	236		1.56
	310	236		1.68

50 mph	349	274	2.2	1.70
	355	274		1.84
	360	274		1.95
	365	274		2.06
	371	274		2.20
55 mph	408	309	2.7	2.24
	413	309		2.36
	418	309		2.47
	423	309		2.59
	428	309		2.70

a. PT: passage time; b. calculated based on the lowest protected speed of 30 mph; c. location is measured from stop line

Moreover, the program computed each vehicle's DCP and calculated the $C_{\text{Hourly-Total}}$ for each simulation run. Note that because each simulation run lasted 10 simulation minutes, the equation of $C_{\text{Hourly-Total}}$, which was originally represented by Equations (7), needs to be modified into the following form:

$$C_{Hourly-Total} = 6 \times \sum_{i}^{10\,\text{min}} DCP_i \tag{12}$$

The average $C_{\text{Hourly-Total}}$ for the 30 simulation runs was computed for each candidate detector configuration for a specific volume condition. Therefore, each candidate detector configuration had five $C_{\text{Hourly-Total}}$ for the five volume conditions. Similarly, the software program also computed the D_{Overall} for each simulation run based on Equation (8). The average D_{Overall} for the 30 simulation runs was computed for each candidate detector configuration for a specific volume condition. Therefore, each candidate detector configuration had five D_{Overall} for the five volume conditions as well. Similarly, the average max-out occurrence percentage for each candidate detector configuration was computed as well.

To convert traffic conflicts and delay into money, the hourly dilemma conflict cost ($Cost_{Conflict-Hourly}$) and the hourly delay cost ($Cost_{Delay-Hourly}$) for each volume condition were computed using Equations (9) and (10). Finally, the daily dilemma conflict cost and the daily delay cost were computed using the following two equations, respectively.

$$\begin{aligned}\sum_{i=1}^{24} Cost_{i-Conflicts-Hourly} &= 5 \times Cost_{VeryLowVol-Conflicts-Hourly} + 5 \times Cost_{LowVol-Conflicts-Hourly} \\ &+ 7 \times Cost_{ModerateVol-Conflicts-Hourly} + 5 \times Cost_{HighVol-Conflicts-Hourly} + 2 \times Cost_{VeryHighVol-Conflicts-Hourly}\end{aligned} \tag{13}$$

$$\begin{aligned}\sum_{i=1}^{24} Cost_{i-Delay-Hourly} &= 5 \times Cost_{VeryLowVol-Delay-Hourly} + 5 \times Cost_{LowVol-Delay-Hourly} \\ &+ 7 \times Cost_{ModerateVol-Delay-Hourly} + 5 \times Cost_{HighVol-Delay-Hourly} + 2 \times Cost_{VeryHighVol-Delay-Hourly}\end{aligned} \tag{14}$$

Where, $\sum_{i=1}^{24} Cost_{i-Conflicts-Hourly}$ = the daily dilemma conflict cost ($);

$Cost_{VeryLowVol-Conflicts-Hourly}$ = hourly dilemma conflict cost under very low traffic ($);

$Cost_{LowVol-Conflicts-Hourly}$ = hourly dilemma conflict cost under low traffic ($);

$Cost_{ModerateVol-Conflicts-Hourly}$ = hourly dilemma conflict cost under moderate traffic ($);

$Cost_{HighVol-Conflicts-Hourly}$ = hourly dilemma conflict cost under high traffic ($);
$Cost_{VeryHighVol-Conflicts-Hourly}$ = hourly dilemma conflict cost under very high traffic ($);
$Cost_{VeryLowVol-Delay-Hourly}$ = hourly delay cost under very low traffic ($);
$Cost_{LowVol-Delay-Hourly}$ = hourly delay cost under low traffic ($);
$Cost_{ModerateVol-Delay-Hourly}$ = hourly delay cost under moderate traffic ($);
$Cost_{HighVol-Delay-Hourly}$ = hourly delay cost under high traffic ($);
$Cost_{VeryHighVol-Delay-Hourly}$ = hourly delay cost under very high traffic ($).

Similarly, the daily average max-out occurrence percentage was also computed for each candidate location of the upstream detector using the following equation.

$$\begin{aligned} P_{Maxout} &= 5 \times P_{Maxout-VeryLowVol} + 5 \times P_{Maxout-LowVol} + 7 \times P_{Maxout-ModerateVol} \\ &+ 5 \times P_{Maxout-HighVol} + 2 \times P_{Maxout-VeryHighVol} \end{aligned} \qquad (15)$$

Where, P_{Maxout} = daily average max-out occurrence percentage (%);
$P_{Maxout-VeryLowVol}$ = max-out occurrence percentage under very low traffic (%);
$P_{Maxout-LowVol}$ = max-out occurrence percentage under low traffic (%);
$P_{Maxout-ModerateVol}$ = max-out occurrence percentage under moderate traffic (%);
$P_{Maxout-HighVol}$ = max-out occurrence percentage under high traffic ($);
$P_{Maxout-VeryHighVol}$ = max-out occurrence percentage under very high traffic ($).

The results of the optimization were based on assessing the daily combined cost of dilemma conflicts and delay. The candidate detector configuration that had the lowest daily combined cost of dilemma conflicts and delay among all the candidates of the speed limit was determined as the optimal one for the specific speed limit. Table 4 summarizes the results of the optimization for the four different speed limits, with the optimal configuration identified for each speed limit.

TABLE 4 Optimization results and optimal configuration for each speed limit.

Speed Limit	Candidate Detector Configuration		Daily Delay Cost ($)	Daily Dilemma Conflict Cost ($)	Daily Combined Cost ($)	Daily Average Max-out Occurrence Percentage (%)
	Upstream Detector	Downstream Detector				
40 mph	243	209	$2,280.30	$47.28	$2,327.58	0.49%
	250	209	$2,289.91	$12.65	$2,302.56	0.45%
	255*	209*	$2,289.71	$7.32	$2,297.03[a]	0.42%
	260	209	$2,303.37	$4.53	$2,307.90	0.55%
	265	209	$2,317.96	$5.30	$2,323.26	0.62%
	270	209	$2,333.40	$5.07	$2,338.47	0.63%
45 mph	289	236	$2,375.00	$34.54	$2,409.54	0.45%
	295	236	$2,372.56	$22.75	$2,395.31	0.53%
	300	236	$2,380.83	$16.62	$2,397.45	0.59%
	305	236	$2,377.16	$13.14	$2,390.30	0.64%
	310*	236*	$2,374.22	$7.53	$2,381.75 [a]	0.68%

50 mph	349*	274*	$2,500.33	$35.61	$2,535.95 [a]	2.68%
	355	274	$2,522.24	$24.96	$2,547.20	3.05%
	360	274	$2,535.05	$22.70	$2,557.75	3.61%
	365	274	$2,540.99	$23.33	$2,564.33	4.22%
	371	274	$2,553.46	$18.52	$2,571.97	4.86%
55 mph	408	309	$2,548.71	$33.75	$2,582.46	3.91%
	413*	309*	$2,550.48	$29.82	$2,580.30 [a]	4.38%
	418	309	$2,567.36	$24.92	$2,592.28	4.52%
	423	309	$2,579.16	$19.98	$2,599.15	5.27%
	428	309	$2,573.70	$21.17	$2,594.87	5.85%

* denotes the optimal detector configuration for the speed limit; a refers to the lowest value in the column for the speed limit.

Evaluation of the Optimal Advance Detector Configurations

In this section, the optimal advance detector configuration was evaluated through comparison with four classic detector configurations, i.e., Beirele, Bonneson, SSITE, and Winston-Salem configurations (2, 16), which have been recommended by FHWA or widely used in the nation.

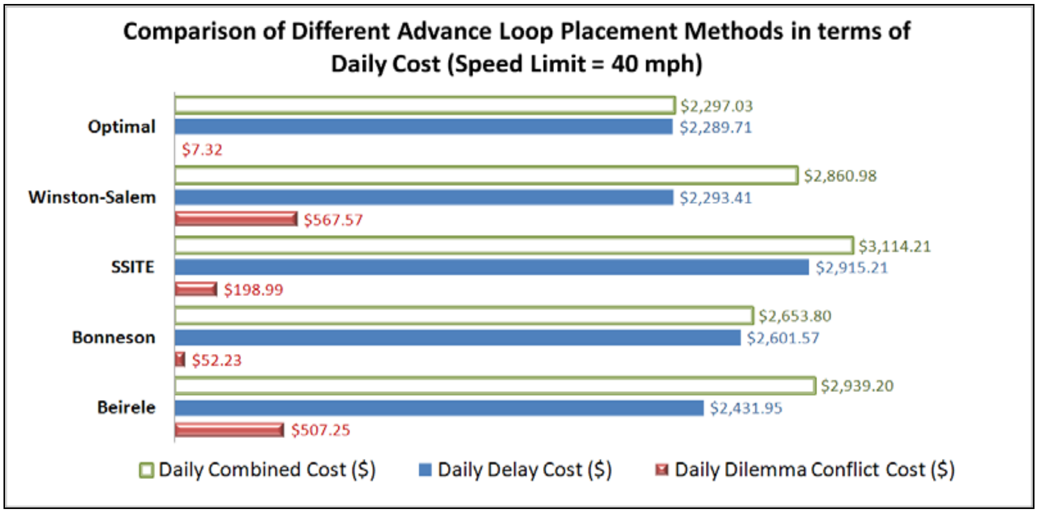
Before the comparison, the four classic configurations were evaluated using the simulation test bed. Microscopic simulation models of these classic configurations were built in VISSIM. Thirty 600-second simulation runs were then performed for each classic configuration under each traffic volume condition summarized in Table 2. The daily dilemma conflict cost, daily delay cost, and daily average max-out occurrence percentage were hence computed for each classic configuration.

Table 5 summarizes the comparison results for speed limits of 40mph and 50mph, while Figure 4 depicts the comparisons using graphic presentation.

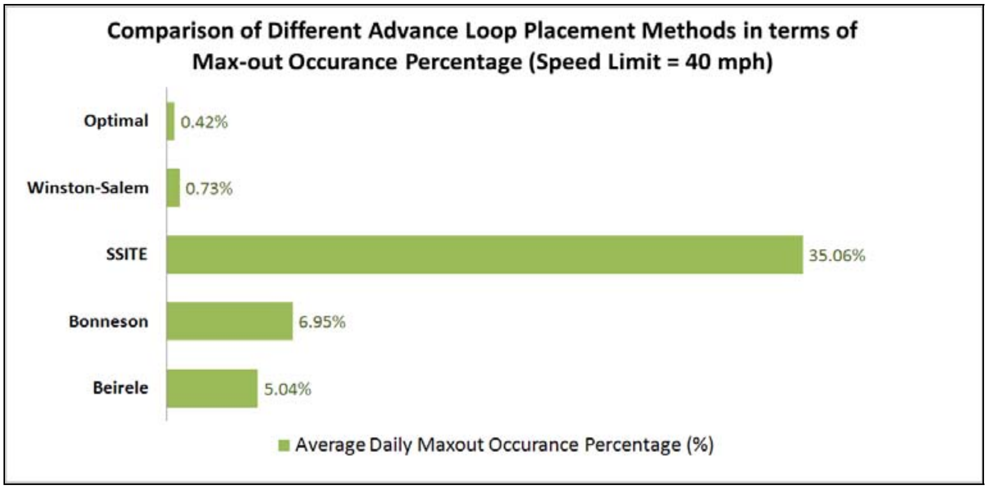
TABLE 5 Comparison between the classic configurations and the optimal configuration

Speed Limit	**Detector Configuration**	**Daily Delay Cost ($)**	**Daily Dilemma Conflict Cost ($)**	**Daily Combined Cost ($)**	**Daily Average Max-out Occurrence Percentage (%)**
40 mph	Beirele	$2,431.95	$507.25	$2,939.20	5.04%
	Bonneson	$2,601.57	$52.23	$2,653.80	6.95%
	SSITE	$2,915.21	$198.99	$3,114.21	35.06%
	Winston-Salem	$2,293.41	$567.57	$2,860.98	0.73%
	Optimal	$2,289.71 [a]	$7.32 [a]	$2,297.03 [a]	0.42% [a]
50 mph	Beirele	$2,571.80	$210.11	$2,781.90	10.03%
	Bonneson	$2,773.40	$44.56	$2,817.97	19.22%
	SSITE	$2,917.84	$98.25	$3,016.09	40.60%
	Winston-Salem	$2,381.34 [a]	$218.59	$2,599.93	3.17%
	Optimal	$2,500.33	$35.61 [a]	$2,535.95 [a]	2.68% [a]

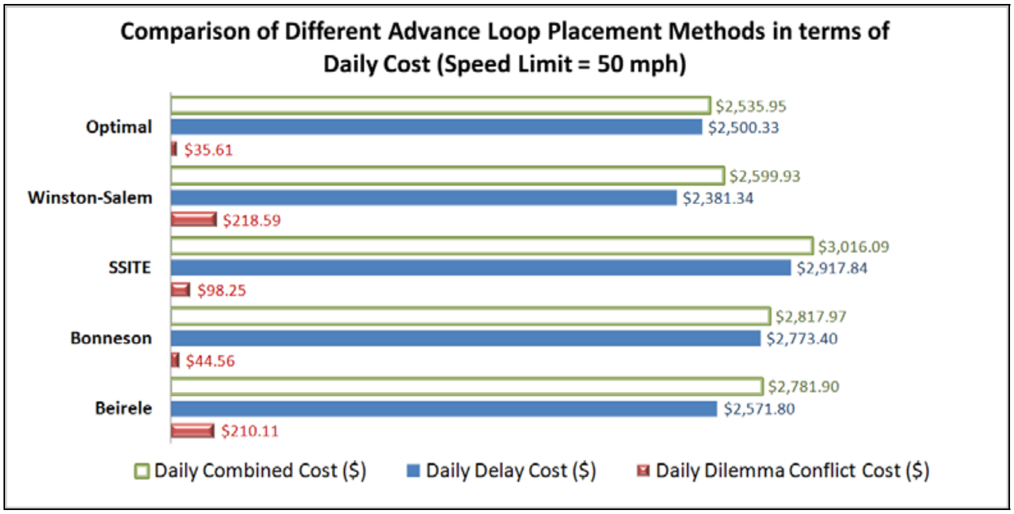
a denotes the lowest value in the column for the speed limit


Comparison of Different Advance Loop Placement Methods in terms of Daily Cost (Speed Limit = 40 mph)
Optimal
$2,297.03
$2,289.71
$7.32
Winston-Salem
$2,860.98
$2,293.41
$567.57
SSITE
$3,114.21
$2,915.21
$198.99
Bonneson
$2,653.80
$2,601.57
$52.23
Beirele
$2,939.20
$2,431.95
$507.25
Daily Combined Cost ($)
Daily Delay Cost ($)
Daily Dilemma Conflict Cost ($)

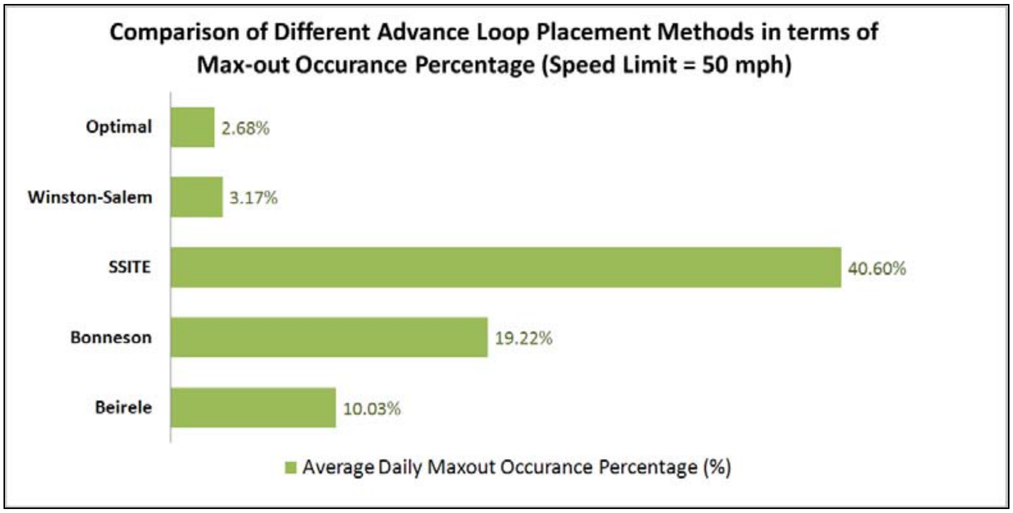
(a)


Comparison of Different Advance Loop Placement Methods in terms of Max-out Occurance Percentage (Speed Limit = 40 mph)
Optimal
0.42%
Winston-Salem
0.73%
SSITE
35.06%
Bonneson
6.95%
Beirele
5.04%
Average Daily Maxout Occurance Percentage (%)

(b)


Comparison of Different Advance Loop Placement Methods in terms of Daily Cost (Speed Limit = 50 mph)
Optimal
$2,535.95
$2,500.33
$35.61
Winston-Salem
$2,599.93
$2,381.34
$218.59
SSITE
$3,016.09
$2,917.84
$98.25
Bonneson
$2,817.97
$2,773.40
$44.56
Beirele
$2,781.90
$2,571.80
$210.11
Daily Combined Cost ($)
Daily Delay Cost ($)
Daily Dilemma Conflict Cost ($)

(c)



(d)

FIGURE 4 Comparison of different advance detector configurations

Under the speed limit of 40 mph, the optimal configuration had the lowest daily dilemma conflict cost. The number was much lower when compared with other classic configurations. This fact revealed that for the 40mph speed limit, the optimal configuration was much safer than the classic configurations. Tied with Winston-Salem configuration, the optimal configuration also had the lowest daily delay cost. It indicated that the optimal configuration was the most operational efficient configuration among all configurations. Moreover, the optimal configuration was also the configuration that had the lowest daily average max-out occurrence percentage. All these ensured the optimal configuration to have the lowest daily combined cost among all configurations.

Under the speed limit of 50 mph, the optimal configuration had much lower daily dilemma conflict cost compared with any of the four classic configurations, which meant the optimal configuration was the safest one among all configurations. For the operational efficiency, the optimal configuration ranked the second behind Winston-Salem configuration by having the second lowest daily delay cost. However, Winston-Salem configuration sacrificed its safety performance to achieve a better operational efficiency, which was reflective of its highest daily dilemma conflict cost among all configurations. Moreover, the optimal configuration was least likely to max out among all configurations. From an overall perspective, the optimal configuration also had the lowest daily combined cost under the speed limit of 50mph.

CONCLUSIONS

The excellent performance of the optimal configuration in the evaluation suggested that the proposed optimization was effective in generating desirable detector configurations that can minimize the combined cost of safety and delay. When compared with any of the four classic configurations, the proposed alternative advance detector configuration had a lower safety cost, a lower combined daily cost, and a lower occurrence rate of max-out. These facts sufficiently validated the alternative advance detector configuration in terms of providing effective and efficient protection to vehicles at high speed signalized intersections.

The achievement of the excellent performance was majorly benefited from the following aspects:

- Designed for option zone protection: the dynamic option zone model was found to estimate the actual dilemma zone most accurately among all available dilemma zone models.
- Safety priority: the design criteria assured that there is no compromise of safety;
- Selection of proper design goal: the design goal was to carry vehicles through the option zone rather than to the stop line, which enhanced the operational efficiency while not compromising any safety; and,
- Based on accurate option zone locations: the option zone model had dynamic contributing factor values, and was developed based on field-collected vehicle trajectory data. It was well reflective of the real-world conditions.

In conclusion, the superiority of the option zone based detector configuration has been proved through this research. Future research will be focused on the optimization of the yellow interval as well as the passage time in order to continuously improve the overall performance of the option zone based detector configurations.

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